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MATHEMATICAL NOTES

EDITED BY DAVID DRASIN

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AND WHAT IS YOUR ERDÖS NUMBER?

CASPER GOFFMAN, Purdue University

The great mathematician Paul Erdös has written joint papers with many mathematicians. This fact may lend some interest to the notion of Erdös number which we are about to describe.

Let A and B be mathematicians, and let A_i, $i=0, 1, \cdots, n$, be mathematicians with $A_0=A$, $A_n=B$, where A_i has written at least one joint paper with A_{i+1}, $i=0, \cdots, n-1$. Then $A_0, A_1, \cdots, A_n$ is called a chain of length n joining A to B. The A-number of B, $\nu(A; B)$, is the shortest length of all chains joining A to B. If there are no chains joining A to B, then $\nu(A; B)=+\infty$. Moreover, $\nu(A; A)=0$. Then $\nu(A; B)=\nu(B; A)$ and $\nu(A; B)+\nu(B; C)\geqq\nu(A; C)$.

For the special case $A=$ Erdös, we obtain the function ν(Erdös; $\cdot$) whose domain is the set of all mathematicians.

I was told several years ago that my Erdös number was 7. It has recently been lowered to 3. Last year I saw Erdös in London and was surprised to learn that he did not know that the function ν(Erdös; $\cdot$) was being considered. When I told him the good news that my Erdös number had just been lowered, he expressed regret that he had to leave London the same day. Otherwise, an ultimate lowering might have been accomplished.

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MATHEMATICAL NOTES

Edited by Robert Gilmer

Manuscripts for this Department should be sent to Robert Gilmer, Department of Mathematics, Florida State University, Tallahassee, FL 32306. Notes are usually limited to three printed pages.

ON THE FUNDAMENTAL PROBLEM OF MATHEMATICS

P. Erdös, Hungarian Academy of Science

I read with interest the paper of C. Goffman, "And What is your Erdös Number?" (this Monthly, 76 (1969) 791). For some time I have considered a problem which I feel is of much more fundamental importance.

We define a graph $\mathfrak{G}(M)$ as follows: the vertices of our graph $\mathfrak{G}(M)$ are the mathematicians. Two vertices are joined if the corresponding mathematicians have written at least one joint paper. (For the time being, let us ignore the papers with more than two authors.)

Is $\mathfrak{G}(M)$ planar, that is, can it be imbedded in E^2? I was not able to solve this interesting and important question. It seems that $\mathfrak{G}(M)$ does not, at present, contain a complete pentagon $\mathfrak{K}(5)$. It certainly contains a $\mathfrak{K}(4)$; for example, in the set Erdös-Rényi-Szekeres-Turan, each pair has a joint paper.

I communicated this problem to Schinzel, who proved that $\mathfrak{G}(M)$ is not planar by showing that $\mathfrak{G}(M)$ contains a $\mathfrak{K}(3,3)$ — that is, a complete bipartite graph of 6 vertices (with three vertices of each color and the 9 edges connecting black to white in all possible ways). The white vertices are Chowla, Mahler, Schinzel; the black ones are Davenport, Erdös, Lewis; the simple task of finding the 9 relevant papers can be left to the reader.

I would like to mention some interesting related problems. There are sets of three mathematicians, each subset of which has a paper (more precisely, only the empty set has no papers); for example, Erdös-Rogers-Taylor. It would be nice to have an example of a set of 4 mathematicians where each of the 15 non-empty subsets has a paper. I believe such a set does not yet exist.

The graph $\mathfrak{G}(M)$, in fact, should be denoted by $\mathfrak{G}^{(t)}(M)$ (t stands for time). I suggest the following optimistic conjecture: to each integer r there is a time t_r so that for $t > t_r$ the graph $\mathfrak{G}^{(t)}(M)$ contains a complete graph $\mathfrak{K}(r)$ of r vertices.