## The Petersen graph is not planar

The following graph is called the *Petersen graph*. We will show that it is not planar by finding a subgraph that is homeomorphic to the complete bipartite graph  $K_{3,3}$ 



First, label the vertices of the Petersen graph as shown. The subgraph that is homeomorphic to  $K_{3,3}$  will have vertex set partitioned as  $\{v_1, v_2, v_3\} \cup \{w_1, w_2, w_3\}$ . Remove the edges  $(x_1, x_2)$  and  $(x_3, x_4)$ , and then remove the vertices  $x_1, \ldots, x_4$  through series reductions.



The result is homeomorphic to  $K_{3,3}$ 

